Admissions Test Syllabus for International Applicants
Main field of study 01.04.01 Mathematics
Educational programme Advanced Mathematics"Advanced Mathematics"

Admissions test form. This admissions test consists of two stages:
1. Competitive selection of documents
(portfolios)
2. An interview
The second stage is only for those applicants who have obtained at least 35 points for the first stage.

Stage 1: The competitive selection of documents (portfolios)

The official documents, certificates and other materials must be submitted in legible copies. If any document is written in a language other than Russian or English, the applicant must submit its translation into one of these languages (translations into Russian must be notarised (translations into Russian have to be notarised by either a notary or the Russian embassy/consulate in the foreign country or the embassy/consulate of the foreign country in Russia unless otherwise specified in the international agreements of the Russian Federation).

Section. 1.1. Portfolio composition: the list of documents and points scored

Documents / documented facts for evaluation		**Assessment criteria**	**The number of points scored**
1	Motivation letter (obligatory)	The criteria are specified below, in Section 1.2 "Motivation letter"	**25**
2	Essay (a research paper) (obligatory)	The criteria are specified in Section 1.3 "Essay"	**10**
3	Curriculum Vitae (CV) containing the complete and continuous history of the applicant's professional activity and education prior to documents submission (obligatory)	The experts evaluate whether the education corresponds to the world standards (taking into account the reputation of the educational institutions in terms of the mathematical sciences, the reputation of the programmes in terms of the presence and number of graduates who have obtained any results of the world level, additional education obtained during summer schools, inter-university exchange programmes, online	**60**

	The CV may also contain an e-mail address of one or two recommenders ready to submit letters of recommendation if requested.	courses and similar events). Diplomas of winners and laureates of international and national student olympiads in the subjects corresponding to the main field of study in accordance with the relevant consolidated group of fields of study, and information on the applicant's publications in the relevant subjects. The requirements and evaluation criteria for letters of recommendation are listed in Section 1.4. Letters of recommendation.	
4	Documents confirming the applicant's English language proficiency (international certificates and other documents). If English is either the applicant's native language or the language of instruction of the previous educational programme, the applicant obtains 5 points automatically. *Only one document is considered. If two or more documents are submitted, the points shall not be summarised.*	English: TOEFL, IELTS, Cambridge CAE, Cambridge CPE, Cambridge FCE – A	5
		Maximum score for the section	**5**
		TOTAL SCORE	**100**

Section 1.2. Motivation letter

up to 25 points

In case any unjustified borrowings are found, the applicant obtains 0 points for his/her motivation letter.

Requirements to the motivation letter formatting and content

1. Font: Times New Roman, point size: 12, line interval: 1,5.
2. The motivation letter must be written in English. It must contain the following:

– information on the applicant's professional training and/or any other activity that may be useful during the period of study in the chosen master's degree programme; information on the success and achievements in the chosen field;

– a well-reasoned substantiation of why the applicant has chosen this particular master's degree programme at St Petersburg University. The applicant must also prove his/her interest in the chosen programme;

– prospects / plans of the obtained knowledge implementation in future professional career.

Evaluation criteria	**Score**
Well-reasoned substantiation of why the applicant has chosen this programme	1
Well-reasoned substantiation of why the applicant has chosen St Petersburg University	1
The presence of description of the competences the applicant is willing to acquire during his/her studies	1
Description of the applicant's academic and practical achievements	From 0 to 15
other information and characteristics the applicant considers relevant (practical experience, basic education, individual abilities and hobbies)	1
Description of the prospects of the obtained knowledge implementation in future professional career.	1
English language skills	From 0 to 5
Maximum points	**25**

Section 1.3. Essay (research paper)

up to 10 points

The applicant personally formulates the title of his/her essay (research paper) within the framework of the field of study of the chosen programme. It is permitted to submit essays based on the applicant's theses (bachelor's, master's, etc.) or scientific publications.

In case any unjustified borrowings are found, the applicant obtains 0 points for his/her essay (research paper).

Requirements to the essay (research paper) formatting and content

1. The essay must be composed in either Russian or English.

2. The total length must not exceed 60,000 printed characters (with spaces), including the bibliography.
3. Font: Times New Roman, point size: 12, line interval: 1,5.
4. Proof links to all of the sources used must be provided.
5. The essay text must be complete and thoroughly structured. It must contain an introduction (where the issue is formulated), the main part (with well-reasoned substantial points), a conclusion (with the author's own conclusions concerning the issue) and a bibliography (not exceeding 2 pages).
6. The author must demonstrate a good knowledge of the subject of research, its conceptual framework, terminology, awareness of the common scientific concepts in the given subject area, understanding of the present-day trends and problems arising in this subject studies.

Evaluation criteria	**Points**
Correspondence of the essay (research paper) topic to the main field of study, correspondence of the text to the formulated topic and applicability of the prospective research	From 0 to 1
Problem statement in accordance with the chosen topic	From 0 to 1
Awareness of the existing scientific concepts related to the chosen range of issues	From 0 to 2
Presence of the individual approach to the problem stated; descriptions of the author's theoretical and pract	From 0 to 5
The structured character of the paper, correct usage of scientific terminology, absence of factual, stylistic and other errors	From 0 to 1

Section 1.4. Letters of recommendation

The letters of recommendation must be sent by the recommender to math.msc@spbu.ru, with a copy to admission@spbu.ru. Each letter must be printed on an official blank of the university. It also must contain the recommender's contact information and his/her signature.

The experts evaluate:
- whether the recommender's scientific results (including those obtained during the 10 previous years) correspond to the world standards,
- the familiarity level between the recommender and the recommendee,
The recommender's evaluation of the recommendee's achievements and opportunities is also taken into account.

Stage 2. Interview

The second stage involves an interview with the applicant. The applicant is offered questions, the answers to which demonstrate his/her culture of mathematical reasoning, the level of his/her training and his/her command of mathematics, his/her logical reasoning ability and the ability to understand new concepts and operate them. The applicant may choose four or more topics out of those listed in Section 2.1. He/she may be asked questions on these topics.
The test is held via videoconferencing in English.

The answer to each question is rated by awarding the applicant a particular number of points. The maximum score for the interview is 100 points.
A list of applicants recommended for enrollment and a waiting list are formed based on the number of points awarded to the applicants for the interview.

Section 2.1. Main topics to check the level of training in mathematics.

Topic 1. Algebra.

Rings, subrings, ideals. Homomorphism theorem. Polynomial ring, Bezout theorem. The factoriality of a polynomial ring over a field. Vector spaces. Linear relationship. The existence of a basis in the vector space. Linear mappings. Rank of linear mapping, Kronecker-Capelli theorem. Eigenvalues and characteristic polynomial. Hamilton-Cayley theorem. Nilpotent operators. Jordan normal form over complex numbers.

Topic 2. Geometry and topology.

Euclidean spaces, scalar product, distances, angles. Affine and orthogonal transformations, movements. Curves and surfaces of the second order. Curvature of a curve on a plane, curvature and torsion of a space curve, Frenet formulas. Metric and topological spaces, continuous maps of topological spaces. Connectivity, linear connectivity, compactness. Homotopies of maps. The fundamental group of topological space. The fundamental group of the circle.

Topic 3. Mathematical analysis and Fourier analysis.

Limits. Compactness. Continuity. Uniform convergence. Differential and derivative. Extrema of functions. Taylor series. Riemann integral.

Differentiable mappings. Conditional extrema. The method of Lagrange multipliers. Lebesgue integral. L^p classes. Tonelli's theorem. Fubini's theorem. Convolution of functions. Holomorphic functions. Cauchy's theorem. Liouville's theorem. Deductions. Rouché's theorem. Fourier series. Dirichlet kernels, Fejér kernels. Decrease of Fourier coefficients. Plancherel theorem.

Topic 4. Ordinary differential equations and mathematical physics.

Existence and uniqueness of solutions. Linear systems of differential

equations. Dependence of solutions on the initial data and parameters. Lyapunov stability. Statement of the main problems of mathematical physics. Solution of differential equations in generalised functions. Fundamental solution and Cauchy problem.

Topic 5. Discrete mathematics.

Graphs, directed graphs, trees, connected components in a directed and an undirected graph. Matchings, Hall's lemma. Planar graphs, Euler's formula. Euler paths and cycles. Permutations, cyclic type. Combinations, combinations with repetitions, placements.

Topic 6. Probability theory.

Probability spaces, distributions of random variables, criteria for the independence of random variables, numerical characteristics of random variables, Bernoulli trials, de Moivre–Laplace local and integral theorems. The law of large numbers and the central limit theorem for sums of independent random variables. Characteristic functions. Markov chains with a finite or countable set of states. Discrete-time martingales.

Topic 7. Mathematical logic and set theory.

The language of propositional classical logic and its two-valued semantics.

Disjunctive normal forms (DNF) and conjunctive normal forms (CNF). Theorem on reducing propositional formulas to DNF and CNF.

Hilbert calculus for propositional classical logic and derivability in it. Deduction theorem for this calculus. Consistent and maximal consistent sets. Strong completeness theorem (including correctness)

Hilbert calculus for propositional classical logic and its most important corollaries.

The paradoxes of the naive set theory. The concept of Zermelo–Fraenkel set theory with the axiom of choice. Basic operations on sets and their basic properties. Ordered pairs, triples, etc. Cartesian products. Relations and functions. Equivalence relations and partial orders. Partially ordered sets (posets). Founding and transfinite induction. Linear posets and their initial segments. Well-ordered sets (wellorders) and transfinite recursion. Proposal for wellorder isomorphisms. Wellorder comparability theorem. Equinumerosity and its simplest properties. Cantor–Schröder–Bernstein theorem.

Power comparability theorem. Cantor's theorem (on the power of the set of all subsets in the given set). Countable sets and their main properties. Powers of a union of sets and product of sets.

Topic 8. Theoretical informatics

The complexity of algorithms in time and methods of its evaluation. The main theorem on the operating time of recursive algorithms (Master theorem). Graph search algorithms (breadth-first search, depth-first search, Dijkstra's algorithm). Sorting algorithms (insert, merge, quick sort, heap sort). Data structures for representing sets (list; AVL tree or red/black tree; hash table), operations on them. Finite state machines (deterministic and non-deterministic), their equivalence. Computational complexity: NP complexity class, examples of nondeterministic polynomial time complete problem. Algorithmically unsolvable problems.

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