Open International Olympiad Guidelines
for foreign students in bachelor's degree programmes
in 2024/25 academic year in the subject
'Modern Mathematics (in English)' at St Petersburg University

The admissions test consists of two stages:
1. Olympiad assignment
2. Interviewing.

Only those applicants who have scored at least 35 points for the first stage shall be admitted to the second stage.
In order to pass the Olympiad, the applicants must score at least 50 points.

STAGE 1: OLYMPIAD ASSIGNMENT

Official documents, certificates and other materials shall be provided in legible copies. Should the language of any document be other than Russian or English, applicants must submit their respective document translated into either of the language (the texts of the translated documents must be notarised by a notary or embassy/consulate of the Russian Federation in a foreign state or by an embassy/consulate of a foreign state in the Russian Federation unless otherwise is provided for by international treaties of the Russian Federation).

Section 1.1. List of documents and scored points

Documents / documented facts to be evaluated		Evaluation criteria	Score of points
1	A motivation letter (mandatory)	The criteria are specified in section 1.2. 'A Motivation Letter'	25
2	Essay (research paper) (mandatory)	The criteria are specified section 1.3. 'An Essay'	10

3	Curriculum Vitae (CV) with the complete and continuous record of profession-associated activities and training by the moment of submitting (mandatory). The CV may contain the e-mail address of one or two reference-givers who will provide their letters of recommendation upon a request.	The admission board experts will evaluate the level of the obtained education in comparison with the world standards (with regard to a respective educational institution reputation in terms of mathematical sciences, reputation of the degree programmes in terms of availability and a quantity of alumni with globally significant scientific achievements, non-degree programmes completed in the framework of summer schools, university exchange programmes, online course, etc.). At the same time, diplomas of international and national Olympiads for university students in the subjects relevant to the topics of the Olympiad subject, as well as information about available relevant publications shall be considered. Requirements to and reference letter assessment criteria are available in Section 1.4 'Reference Letters'.	60
4	Documents confirming the English language fluency (international certificates and other documents). If English is the applicant's native language or the language of instruction of the previous educational programme, the applicant shall score five (5) points by default. Accepted can be only one document. Should there be two or more certificates, the points shall not be summed	English language reference credentials: TOEFL, IELTS, Cambridge CAE, Cambridge CPE, Cambridge FCE – A	5
		TOTAL SCORE OF POINTS	**100**

Section 1.2. Motivation letter
up to 25 points

Should any illegal borrowings be detected in the paper, the motivation letter shall be scored by zero (0) points.

Requirements to the motivation letter content and design

The motivation letter shall be provided in English and contain:

- information about the applicant's professional training/activity, correspondent to the topics of the Olympiad subject, information about promotion and achievements in the chosen field;
- justified reasons for choosing the given Olympiad subject, the proof of th applicant's interest in the Olympiad subject;
− prospects/plans to implement the obtained knowledge within the future professional career.

Evaluation criteria	**Score of points**
a well-reasoned argumentation of the Olympiad subject choice	1
a well-reasoned argumentation of the choice to participate in the St Petersburg University Olympiad	1
indication of the competences that the applicant intents to acquire in the future	1
indication of the applicant's academic and practice-oriented achievements	from 0 to 15
other information and details that the applicants have indicated at their discretion (practice-oriented experience, background education, individual abilities and hobbies)	1
prospects to implement obtained knowledge in the future professional career	1
English language fluency	from 0 to 5
Maximum score of points	**25**

Section 1.3. Essay (research paper)
up to 10 points

The applicant shall independently formulate the essay (research paper) topic within the chosen field of Olympiad subject. Applicants can use or refer to their graduation projects (bachelor's degree, master's degree, etc.) or scientific papers for their essays.

Should any illegal borrowings be detected in the paper, the essay (research paper) shall be scored by zero (0) points.

Requirement to the essay (research paper) content and design

1. the essay should be provided either in Russian or in English.
2. The total count of the text shall not exceed 60,000 printed characters (with spaces), including the reference list.
3. Applicant must provide links to all reference resources.
4. The text of the essay must be a complete and thoroughly structured one and contain an introduction (where the issue is formulated), the main part (with well-reasoned substantial points on the issue), a conclusion (with the author's own conclusions on the issue concerned) and a reference list (not exceeding 2 pages).
5. The author must demonstrate a deep insight into the subject of research, its conceptual apparatus, terminology, awareness of the commonly accepted scientific concepts in the given subject area, understanding of the present-day trends and problems arising throughout this subject studies.

Evaluation criteria	**Points**
The theme of the essay (research paper) correlates with the Olympiad subject; the discourse of the paper addresses the topic; the topic is relevant	from 0 to 1
Stating the issues within the chosen topic	from 0 to 1
Knowing current scientific concepts on the respective issues	from 0 to 2
Presence of the author's independent approach to the stated issues, availability of the author's theoretical and practical developments	from 0 to 5
The structure of the paper, appropriate use of scientific terminology, absence of factual, stylistic and other errors.	from 0 to 1

Section 1.4. Reference Letters

The recommenders are requested to forward their letters of recommendation to math.msc@spbu.ru with a c/c admission@spbu.ru. The recommenders are advised to use their respective university letter-head with the reference-givers' contact details and their handwritten signature.

The admission board experts shall consider:

- whether recommender's scientific achievements (during ten (10) previous years) meet world standards;
- the degree of acquaintance between the applicant and the recommender.

Also, the board shall consider the recommender's evaluation of the applicant's achievements and potency emphasised in the letter of recommendation.

STAGE 2: INTERVIEWING

At the second stage applicants shall be interviewed. In the course of the interview, the applicants shall be interviewed on the questions that will demonstrate their mathematical culture, the level of their proficiency and operational fluency in the respective material, the ability to think logically, as well as to understand new concepts and refer to them. The applicants have the right to select as little as four (4) topics from Section 2.1, within which they will be interviewed.

The interview is conducted using video conferencing in English.

For each answer, the applicant is conferred a particular score of points. The maximum possible score of points for the interview is 100.

Based on the applicants' score of points for the interviews, the board shall draw up an admission list and the ranked waiting list.

Section 2.1 Basic topics to evaluate the level of proficiency in mathematics

Topic 1. Algebra:

Rings, subrings, ideals. The homomorphism theorem. Ring of polynomials, Bezout's theorem. Factoriality of a polynomial ring over a field. Vector spaces. Linear dependency. Existence of a basis in a vector space. Linear mappings. Rank of a linear mapping, Kronecker-Capelli theorem. Eigenvalues and characteristic polynomial. Hamilton-Cayley theorem. Nilpotent operators. Jordan normal form over complex numbers.

Topic 2. Geometry and Topology

Euclidean spaces, dot product, distances, angles. Affine and orthogonal transformations, movements. Curves and surfaces of the second order. Curvature of a curve on a plane, curvature and torsion of a spatial curve, Frenet's formulas. Metric and topological spaces, continuous mappings of topological spaces. Connectedness, linear connectivity, compactness. Homotopy mappings. The fundamental group of a topological space. The fundamental group of the circle.

Topic 3. Mathematical analysis and Fourier analysis.

Limits. Compactedness. Persistence. Uniform convergence. Differential and derivative. Function extremes. Taylor series. The Riemann integral.

Differentiable mappings. Conditional extremes. Method of Lagrange multipliers. The Lebesgue integral. Classes L^p. Fubini's theorem.

Fubini’s theorm. Function convolution. Holomorphic functions. Cauchy's theorem. Liouville's theorem. Residues. Rouché's theorem. Fourier series. Nuclei Dirichlet, Fejer. Decreasing Fourier coefficients. Plancherel's theorem.

Topic 4. Ordinary Differential Equations and Mathematical Physics

Existence and uniqueness of solutions. Linear differential systems
equations. Dependence of solutions on initial data and parameters. Stability according to Lyapunov. Statement of the main problems of mathematical physics. Solution of differential equations in generalized functions. Fundamental solution and the Cauchy problem.

Topic 5. Discrete Mathematics

Graphs, directed graphs, trees, connected components in a directed and undirected graph. Matchings, Hall's lemma. Planar graphs, Euler's formula. Euler paths and cycles. Permutations, cycle type. Combinations, combinations with repetitions, placements.

Topic 6. Probability Theory

Probability spaces, distributions of random variables, independence criteria for random variables, numerical characteristics of random variables, Bernoulli tests, local and integral theorems of Moivre-Laplace. The law of large numbers and the central limit theorem for sums of independent random variables. Characteristic functions. Markov chains with a finite or countable set of states. Martingales with discrete time.

Topic 7. Mathematical Logic, Algebra and Number Theory

The language of propositional classical logic and its two-valued semantics Disjunctive normal forms (DNF) and conjunctive normal forms (CNF). The theorem on reduction of propositional formulas to DNF and CNF Hilbert calculus for propositional classical logic and derivability in it. The deduction theorem for this calculus. Consistent and maximal consistent sets. The theorem on strong completeness (including correctness) of the Hilbert calculus for propositional classical logic and its most important consequences.

Paradoxes of Naive Set Theory. An idea of Zermelo–Fraenkel set theory with the axiom of choice. Basic operations on sets and their main properties. Ordered pairs, triples, etc. Cartesian products. Relations and functions. Equivalence relations and partial orders. Partially ordered sets (posets). Foundation and transfinite induction. Linear sets and their initial segments. Well-ordered sets and transfinite recursion. Proposition on well-ordered sets isomorphisms. Well-ordered sets comparability theorem. Equivalence and its simplest properties. The Cantor-Schroeder-Bernstein theorem.

Theorem on comparability of powers. Cantor's theorem (on the cardinality of the set of all subsets of a given set). Countable sets and their main properties. Cardinalities of union sets and product of sets.

Topic 8. Theoretical Informatics

Time complexity of algorithms and methods for its estimation. The main theorem on the running time of recursive algorithms (Master theorem). Graph search algorithms (breadth-first search, depth-first search, Dijkstra's algorithm). Sorting algorithms (insert, merge, "quick sort", heap sort). Data structures for representing sets (list; AVL tree or red-black tree; hash table), operations on them. Finite automata (deterministic and non-deterministic), their equivalence. Computational complexity: NP complexity class, examples of NP-complete problems. Algorithmically unsolvable problems.

SECTION 2.1.1. LIST OF READING

Vinberg, E. *Algebra course*. 4th edition. Moscow. MCNMO. 2011. Chapters 1 - 3, 5 - 6.

Professor of the Department of Russian and World Literatures *Introduction to Algebra.* Part I. Fundamentals of Algebra. 3rd edition. Moscow. Phizmatlit. 2004. Chapters 4 - 5.

Professor of the Department of Russian and World Literatures *Introduction to Algebra.* Part II Linear Algebra. 3rd edition. Moscow. Phizmatlit. 2004. Chapters 1 - 2.

Postnikov, M. *Lectures on Geometry. Semester I. Analytic Geometry.* 2nd edition. Moscow. Nauka. 1986. Chapters 1 - 7, 12 - 13, 16 - 18, 23 - - 26.

Pogorelov A. *Differential Geometry.* 6th edition. Moscow, Nauka. 1974. Chapters 1 - 3.

Borisovich, Yu., Bliznyakov, N., Izrailevich, Ya., Fomenko, T. *Introduction to Topology.* 2th edition. Moscow. Nauka. 1995. Chapters 1 - 3.

Zorich, V., *Mathematical Analysis*. Moscow. MCNMO. 2012. Part 1. Chapters 6 - 8. Part 2. Chapters 9 -13 and 16 - 19.

Doctor of Philology *Course in Differential and Integral Calculus.* St Petersburg. Lan. 2009 Volume 1, Chapters 1 - 4.

Filippov, A. *Introduction to the Theory of Differential Equations.* 2nd 2. MIR, 2007. Chapters 2, par. 5,7. Chapters 3, par. 9, 10, 11, 14. Chapters 4, par. 18. Chapters 5, par. 23.

V.A. Emelichev, V. Melnikov,O., Sarvanov, V., Tyshkevich, R. *Lectures on Graph Theory.* Moscow. Nauka. 1990. Chapters 1, 2, 4, 6.

at St Petersburg University V. Vilenkin, N. *Combinatorics.* Moscow. Nauka. 1969. Chapters 1, 2.

A.N. A. Probability. Books 1 and 2. Moscow. MCNMO. 2004. Chapters 2, 3, 7 (par. 1 - 4, 8). Shen, A. *Languages and Calculus.* 4nd edition, amended. MCNMO. 2012. Sections 1.1, 1.2, 2.1, 2.2, +

K.Gander Mostovsky, A. *Set Theory.* MIR, 1970. Par. 1 - 6, Chapter I, par. 1 - 3, Chapter II+

Vereschagin, N., Shen, A. *Beginnings of Set Theory.* 4nd edition, added. MCNMO. 2012. Sections 1.1, 1.3 - 1.7, 2.1 - 2.6, 2.8,

Vladimirov, V., Zharikov, V. *Equations of Mathematical Physics.* Phizmatlit. 2004. Chapter 2, par. 1.2, 1.4. Chapter 3, par. 3.1, 3.3, 3.5.

Cormen, T., Leiserson, Ch., Rivest, R., Stein, K. *Introduction to Algorithms.*

Williams. 2018. Hopcroft, J., Motwani, R., Ullman, J., *Introduction to Automata Theory, Languages and Computation.* Williams. 2016.